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STRESS CONCENTRATION NEAR AN INHOMOGENEITY AND EXPERIMENTAL CLARIFICATION OF THE COUPLE-STRESS EFFECT

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Following the fundamental papers [1-3], many authors have recently exerted considerable efforts to develop a new mechanics of the microcontinuum in which displacements of microelements are taken into account. To apply these theories it is required to determine a sufficiently large quantity of new elastic constants. The effects which appear because of taking account of the microelement displacements have been examined theoretically by a number of authors. However, there is quite little experimental work in which the effect of this accounting would be explained and new elastic constants would be determined in materials [4-6]. The purpose of the present paper is to clarify the effect of the influence of couple stresses on stress concentration near an inhomogeneity in the case of plane strain by the experimental method of photoelasticity.

Stress concentration near a circular cylinder of radius a exactly coincident with a cavity and imbedded in an infinite medium (strip), which is subject to the action of a uniform load p at infinity, is considered. The circular cylinder (inhomogeneity) and the external medium have different elastic constants: the shear modulus and Poisson ratio G_1, ν_1 and G_2, ν_2; l_1 and l_2 (new elastic constants introduced by the couple-stress theory of elasticity), respectively.

Let us take an r, θ polar coordinate system. Let σ_r, σ_θ, $\tau_{r\theta}$, $\tau_{\theta r}$ denote the stress tensor components and μ_r, μ_θ, the couple-stress tensor components.

Taking the stress function in the form from [2] and forming the boundary conditions of complete contact on the contour of the inhomogeneity (at $r = a$), we find the solution for stresses according to the couple-stress theory of elasticity [1, 2], which is obtained in closed form in terms of the modified Bessel functions I and K. According to the classical theory in which couple-stresses are neglected, the solution is obtained by a passage to the limit when $l_1 = l_2 = 0$.

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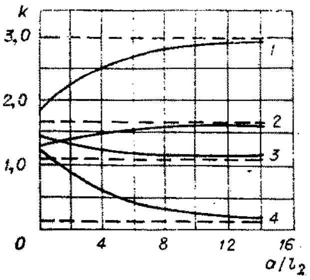

Fig. 1

In the general case, the location of points at which the greatest stresses originate and the magnitude of the stress concentration coefficient are complex functions of the elastic constants G_i, ν_i, l_i $(i = 1, 2)$. The greatest stresses σ_θ develop at $r = a$ and $\theta = \pm\pi/2$. For the values $G_1/G_2 < 1$ the stress concentration coefficient is $k = \sigma_\theta^{(2)}(a, \pm\pi/2)/p$ and for $G_1/G_2 > 1$, $k = \sigma_\theta^{(1)}(a, \pm\pi/2)/p$, where the superscripts (1) and (2) refer to the inhomogeneity and the surrounding medium, respectively:

$$\sigma_\theta^{(1)}(a, \pm\pi/2) = \frac{(1-\nu_2)p}{1+(1-2\nu_1)g} + \frac{2(1-\nu_2)p}{Q}[M + 24(1 - g\nu_1)(l_2/a)^2 L_2(2 - L_1)];$$
$$\sigma_\theta^{(2)}(a, \pm\pi/2) = \frac{\nu_2 + (1-2\nu_1)g}{1+(1-2\nu_1)g}p - \frac{2p}{Q}[(\nu_2 - g)M - 3g(1-\nu_1)N_2L_2(2 - L_1)], \quad (1)$$

where

$$g = G_1/G_2;\ N_1 = 8(1-\nu_1)(l_1/a)^2;\ N_2 = 8(1-\nu_2)(l_2/a)^2;$$

$$L_1 = (a/2l_1)I_1(a/l_1)/I_2(a/l_1);\ L_2 = (a/2l_2)K_1(a/l_2)/K_2(a/l_2);$$
$$Q = g(1-g)(l_2/l_1)^2 N_2 L_2\left[3N_1L_1 - 6N_1 + \frac{4g(1-\nu_1)}{1-g}\right] - $$
$$- [3 - 4\nu_2 - N_2L_2 + g(1 + N_2L_2)]M; \quad (2)$$
$$M = [4g(1-\nu_1)/(1-g)][1 - L_1 - g(l_2/l_1)^2(1 + L_2)] + $$
$$+ (2 - L_1) - [1 + g(l_2/l_1)^2(1 + L_2)](1 + 3N_1L_1 - 6N_1).$$

The dependence of the coefficient k on the ratio a/l_2 obtained from (1) and (2) is shown in Fig. 1 for the following values of the parameters: curve 1) $G_1/G_2 = 0$, $\nu_2 = 1/3$; 2) $G_1 = 0.5G_2$, $\nu_1 = \nu_2 = 1/3$, $l_1 = 0$; 3) $G_1 = 2G_2$, $\nu_1 = \nu_2 = 1/3$, $l_1 = 0$; 4) $G_1/G_2 = \infty$, $\nu_2 = 1/3$. The solid lines correspond to the couple-stress theory and the dashes, to the classical theory.

A polarization optical method was used for an experimental determination of the stress concentration. The strip specimens were fabricated from the epoxy resin ÉD-6, the limit of material proportionality was 530 kg/cm^2, the elastic modulus was 37,500 kg/cm^2, and the Poisson ratio was $\nu = 0.38$. There were three to five small holes of different radius in each specimen. The experiment was conducted for two limit cases: a) $G_1/G_2 = 0$; b) $G_1/G_2 = \infty$, steel disks were hence glued into the holes by cold-hardening epoxy glue.

The greatest normal stress $\sigma_{\theta\max}$ on the hole contour in these limit cases can be calculated [4, 5] from the maximum order of the interference fringes $n_{\max}$ by using the fundamental law of photoelasticity in the form

$$\sigma_{\theta\max} = Cn_{\max}, \quad (3)$$

where C is a constant of the specimen material of given thickness.

A specimen having several small holes of different radius was tested. The concentration coefficients for any holes i and j are hence connected by the relationship

$$k_i/k_j = \Delta(n_{\max})_i/\Delta(n_{\max})_j, \quad (4)$$

where $\Delta(n_{\max})$ is the increment of isochromatic orders on the contour which corresponds to the increment in the applied load.

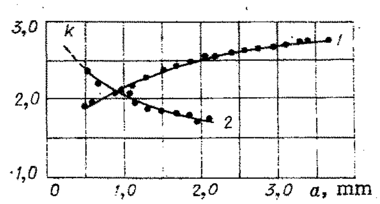

Fig. 2

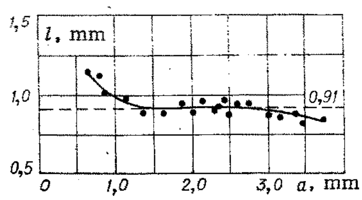

Fig. 3

The high accuracy of this comparison method resides in the use of (4), whereupon the errors due to edge effects, extrapolation, and other factors can be reduced to a minimum.

In this investigation the difference in the increments of the maximum isochromatic order for holes of different radius was found by graphical extrapolation using circular polarization with a magnified image of the interference fringes. Moreover, different holes in one specimen with isochromatic fringe patterns around them were magnified to the same radius, and then these patterns were compared. To determine the position of the orders and the boundary observations an optical comparator was used. Mainly monochromatic light with a 546.1-μm wavelength was used.

Shown in Fig. 2 is an empirical dependence of the stress concentration coefficient k on the hole radius for the same strip material [curve 1) $G_1/G_2 = 0$; 2) $G_1/G_2 = \infty$].

Setting the theoretical and experimental values of the stress concentration coefficient equal, the value of the elastic constant l can be determined. This determination was made for the case $G_1/G_2 = 0$ and its results are presented in Fig. 3, where it is seen that the values of l depend on the magnitude of the hole radius in a number of cases (curved line). The straight line parallel to the horizontal axis corresponds to the ideal case; the values of l are independent of the hole radius. The greatest discrepancies are obtained in the case of small holes; there are some disagreements for comparatively large holes. An abrupt increase in l in the region of quite small holes is explained by the fact that the size of the hole and the thickness of the strip become quantities of the same order of magnitude. The application of couple-stress theory to a specific state of stress requires that the ratio a/l be sufficiently large compared to one (but not greater than 10), as is indicated in [2]. As regards some diminution in l in the domain of large holes, this is apparently explained by the fact that the boundary conditions for a finite-width strip, which are not satisfied completely in the analytical solution by means of couple-stress theory of elasticity, start to be felt.

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